Benha University

Faculty of Commerce

On Characterization of Probability Distributions

By

Ahmed Zakarya Mohammad Afify

Supervised By

Prof. Ahmed Fathy Mahmoud

Department of Statistics, Mathematics and Insurance

Faculty of Commerce

Benha University

Prof. Abdul Hadi Nabih Ahmed

Department of Mathematical Statistics

The Institute of statistical Studies and Research

Cairo University

Dr. Zohdy Mohammed Nofal

Department of Statistics, Mathematics and Insurance

Faculty of Commerce

Benha University

A Thesis Submitted to the Faculty of Commerce, Benha University, In Partial Fulfillment of the Requirements for the Degree of Master of Science in Statistics

2013

Acknowledgments

I would like to express my sincere thanks and gratitude to Almighty Allah for his blessings.

I am indebted and very grateful to my supervisor Professor Ahmed Fathy Mahmoud, for his valuable contributions throughout the research effort, for his great help encouragement, working long hours with me and for his support throughout the research effort.

I would also like to express my greatest debt and gratitude to my supervisor Professor Abdul- Hadi Nabih Ahmed for stimulating my interest in research, for always being available and for his valuable contributions throughout the research effort. Without his guidance, insight, encouragement and willingness to work long hours with me, this thesis would not have been possible.

I am also indebted and very grateful to my supervisor doctor Zohdy Mohammed Nofal for his great help, working long hours with me and for his support.

I wish to thank my professors and the staff of the faculty of commerce, Benha University. Moreover, I wish to express my sincere appreciation to my father, my mother, my brothers, my wife Hend and my son Mohammad whose support and encouragement were vital to the achievement of this thesis.

List of Abbreviations

rv	: $random\ variable$
pdf	: $probability\ density\ function$
df	: $distribution\ function$
Sf	: $survival\ function$
HR	: $hazard\ rate$
MRL	: $mean\ residual\ life$
VRL	: $variance\ residual\ life$
RHR	: $reversid\ hazard\ rate$
RLF	: $residual\ life\ function$
ITF	: $inactivity\ time$
MIT	: $mean\ inactivity\ time$
LMP	: $lake\ of\ memory\ property$

Table of Contents

Acknowledgments ii

List of abbreviations.....iii

Chapter 1 **Research Problem and Review of Literature**

1.1 Introduction……1

1.2 Basic concepts in Reliability ……3

1.2.1 Hazard rate ……..3

1.2.2 Reversed hazard rate……3

1.2.3 Mean residual Life ……4

1.2.4 Vitality function ……5

1.2.5 Variance residual life ……..5

1.2.6 Mean inactivity time ……..6

1.3 Motivation…… 6

1.4 Objectives and Contributions of the Thesis 7

1.5 Characterizations ……7

1.6 Organization of the thesis……..12

Chapter 2 **Characterization of Gamma Distribution by the Truncated Conditional Moments from the Right**

2.1 Introduction ……..13

2.2 Main Results ……14

Chapter 3 **Characterization of Gamma Distribution by the Truncated Conditional Moments from the Left**

3.1 Introduction ……23

3.2 Main Results ……23

Chapter 4 **Characterization of Probability Distributions By The Truncated Moments of Order Statistics**

4.1 Introduction ……30

4.2 Characterization of the Exponential Distribution ……32

Main Results ……32

4.3 Characterization of the Pareto distribution of the first kind ……38

4.3.1 Main Results ……38

4.4 Characterization of the power function Distribution ……42

4.4.1 Main Results ……42

4.5 Characterization of the Pareto distribution 0f the second type ……49

4.4.1 Main Results……49

Chapter 5 **Study Application**……56

APPENDIX I ……..60

APPENDIX II ……..61

References 64